Re-spacing Circles in Existing Rhinestone Patterns[1]

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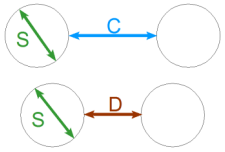
A. Formulas used in the Rhinestone Re-spacing Calculator

The process of re-spacing circles in an existing pattern is a two step process:

(1) Either expand or contract the circles to change the spacing between them using an outline or inline function (also called contouring in some vector programs). The amount to use in this process is calculated as:

$$\text{Outline/Inline Factor} = \frac{|C - D|\ S}{2\,(S + D)}$$



Current Spacing

Desired Spacing

(Or Desired Spacing might be larger than Current Spacing)

Where:

S = Size of the Circles in the Pattern
C = Current Spacing of Circles in the Pattern
D = Desired Spacing of Circles in the Pattern

Note: $|C - D|$ denotes the absolute value of the difference between C and D
If Desired Spacing, D, is smaller than Current Spacing, C (i.e. IF $(D - C) < 0$, then use Outline Function
If Desired Spacing, D, is larger than Current Spacing, C (i.e. IF $(D - C) > 0$, then use Inline Function

(2) After this first step, the circles are now a new size and must be resized back to the original diameter. This is done using a basic scaling process. The amount of scaling needed is calculated as:

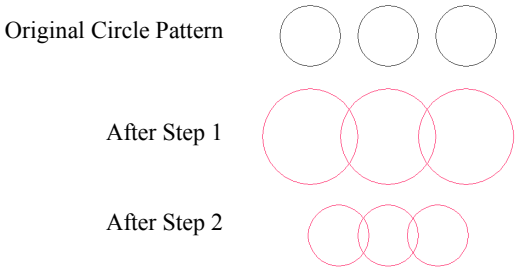
$$\text{Resizing Percentage} = \frac{(S + D)}{(S + C)} \times 100$$

(Note: Because KNK and ACS Studio rescale using a %, this formula multiplies by 100. For programs that re-scale simply by using a ratio, the 100 can be omitted from the formula.)

B. Calculations Used to Arrive at Formulas

When working through the algebra involved, I decided to first work with the case where the spacing will be made smaller. Thus the circles in the pattern will be expanded. Then, after developing the formula, I will then show the other case where the spacing will be made larger.

So, when I first tested this, I decided to just use the entire difference between the Current Spacing and the Desired Spacing for the amount of Outline to apply:



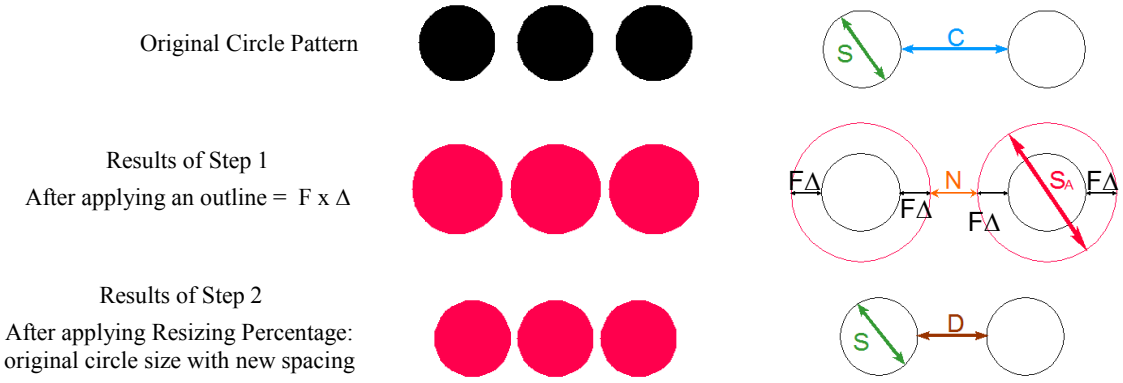
Obviously using the full difference between the Current Spacing and the Desired Spacing is too much. So, I assumed there must be some fraction that must be multiplied by the difference between the two spacings to get the perfect end result. In other words, the correct amount to use in the Outline function would be:

$F \times \Delta$

Where:

F = Rhinestone Spacing Factor
Δ = Difference between Current Spacing and Desired Spacing = C - D

Then the two step process would look like this:



Note the middle diagram on the right shows how the circles are expanded on BOTH sides by F x Δ. And the spacing has been made much smaller:

S_A= Size of Circles After Outline/Inline Function is applied
= S + 2ΔF

N = New spacing after Outline/Inline Function is applied
= Original Spacing, C, minus how much larger the circles were expanded
= C - 2ΔF

Then, in Step 2, the overall pattern is resized so that the circles are back to the original size in the pattern. A scaling ratio function is applied:

$$\text{Scaling Ratio Amount} = \frac{S}{S_A}$$

This scaling amount will also directly affect the new spacing, N, and if we have used the correct Rhinestone Spacing Factor, then our final spacing will equal our Desired Spacing, D. Thus:

$$D = \frac{S}{S_A} \times N$$

$$D = \frac{S}{S + 2\Delta F} \times (C - 2\Delta F)$$

$$D = \frac{SC - 2S\Delta F}{S + 2\Delta F}$$

$$D(S + 2\Delta F) = SC - 2S\Delta F$$

$$DS + 2D\Delta F = SC - 2S\Delta F$$

$$2D\Delta F + 2S\Delta F = SC - DS$$

$$2\Delta F\,(D + S) = S\,(C - D)$$

$$F = \frac{S\,(C - D)}{2\Delta\,(D + S)}$$

Substitute Δ for $(C - D)$

$$= \frac{S\cancel{\Delta}}{2\cancel{\Delta}(D + S)}$$

Δ can be deleted from top and bottom

$$= \frac{S}{2\,(D + S)} \quad \text{OR} \quad \frac{S}{2\,(S + D)}$$

Thus, when using the Outline function, the amount to enter into the settings window:

Outline Factor $= \Delta F$

$$= \frac{(C - D)\ S}{2\,(S + D)}$$

The resizing % needed after performing the outline, to return the circles to their correct size and obtain the correct spacing, is:

$$\text{Resizing \%} = \frac{S}{S_A} \times 100$$

$$= \frac{S}{S + 2\Delta F} \times 100$$

Divide numerator and denominator by S:

$$= \frac{1}{1 + \frac{2\Delta F}{S}} \times 100$$

$$= \frac{1}{1 + \frac{2(C - D) \times S}{S \times 2(S + D)}} \times 100$$

Cross out 2S from top and bottom:

$$= \frac{1}{1 + \frac{(C - D)}{(S + D)}} \times 100$$

$$= \frac{1}{\frac{S + D + C - D}{(S + D)}} \times 100$$

$$= \frac{(S + D)}{(S + C)} \times 100$$

Now we're left with the case of how this works if you need to go the opposite way… to contract the circles; i.e. if you need to expand the spacing. In this case, the circles are smaller by the value placed in the Inline setting and the new spacing will be larger by that amount:

S_A= Size of Circles After Outline/Inline Function is applied
= S - 2ΔF

N = New spacing after Outline/Inline Function is applied
= Original Spacing, C, minus how much larger the circles were expanded
= C + 2ΔF

The same scaling ratio function will be applied:

$$\text{Scaling Ratio Amount} = \frac{S}{S_A}$$

This scaling amount will also directly affect the new spacing, N, and if we have used the correct Rhinestone Spacing Factor, then our final spacing will equal our Desired Spacing, D. Thus:

$$D = \frac{S}{S_A} \times N$$

$$D = \frac{S}{S - 2\Delta F} \times (C + 2\Delta F)$$

$$D = \frac{SC + 2S\Delta F}{S - 2\Delta F}$$

$$D(S - 2\Delta F) = SC + 2S\Delta F$$

$$DS - 2D\Delta F = SC + 2S\Delta F$$

$$-2D\Delta F - 2S\Delta F = SC - DS$$

$$-2\Delta F (D + S) = S (C - D)$$

$$-F = \frac{S (C - D)}{2\Delta (D + S)}$$

Substitute Δ for (C - D), and move negative to other side of the equation

$$F = \frac{-S\ \Delta}{2\ \Delta (D + S)}$$

Δ can be deleted from top and bottom

$$= \frac{-S}{2\ (D + S)} \quad \text{OR} \quad \frac{-S}{2\ (S + D)}$$

Thus, when using the Inline function, the amount to enter into the settings window:

Inline Factor $= \Delta F$

$$= \frac{-(C - D)\ S}{2\ (S + D)}$$

$$= \frac{(D - C)\ S}{2\ (S + D)}$$

The Resizing Percentage is then calculated, in this case, as

$$\text{Resizing \%} = \frac{S}{S_A} \times 100$$

$$= \frac{S}{S - 2\Delta F} \times 100$$

Divide numerator and denominator by S:

$$= \frac{1}{1 - \frac{2\Delta F}{S}} \times 100$$

$$= \frac{1}{1 - \frac{2\,(D - C)\ x\ S}{S\ x\ 2\,(S + D)}} \times 100$$

Cross out 2S from top and bottom:

$$= \frac{1}{1 - \frac{(D - C)}{(S + D)}} \times 100$$

$$= \frac{1}{\frac{S + D - D + C}{(S + D)}} \times 100$$

$$= \frac{(S + D)}{(S + C)} \times 100$$

This is the same formula as in the first case.

Back to the formula for F in both cases:

Outline Factor $= \Delta F$

$$= \frac{(C - D)\ S}{2\,(S + D)}$$

Inline Factor $= \Delta F$

$$= \frac{-(C - D)\ S}{2\,(S + D)}$$

Note in the Inline Factor case, since C is smaller than D (where we want the new Desired Spacing to be larger than the Current Spacing), the calculated value of the Inline Factor will still be a positive number. Thus, for the purpose of automating a formula to calculate the Outline/Inline Factor correctly in both cases, we can use the absolute value of (C - D):

$$\text{Outline/Inline Factor} = \frac{|C - D|\ S}{2\,(S + D)}$$

In programs where Inline is NOT a separate function from Outline, it will probably be necessary to use a negative value for the Inline Factor in order to contract the circles versus expand them.